# Analysis 2

Friday, 7 May 2024

Take one **white** small paper and
one **blue** small paper.

# Vocabulary so far

| | |
|---|---|
| Differential equation (DE) | równanie różniczkowe |
| Ordinary differential equation (ODE) | rów. róż. zwyczajne |
| Partial differential equation (PDE) | rów. róż. cząstkowe |
| First-order, second-order, etc. | pierwszego rzędu, itd. |
| Explicit solution or solution | rozwiązanie jawne |
| Implicit solution | rozwiązanie niejawne |
| General solution | rozwiązanie ogólne |
| Particular solution or specific solution | rozwiązanie specjalne |
| Initial value or initial condition (IC) | warunek początkowy |
| Initial value problem (IVP) | zagadnienie początkowe |

# Separable

Last week

A direct first-order ODE for $y(x)$ can be written in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x).$$

This is solved by justing doing an anti-derivative: $y = \int f(x)\,\mathrm{d}x$ gives an explicit solution for $y$.

An autonomous first-order ODE for $y(x)$ can be written in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = g(y).$$

This is solved by first "separating variables". The equation $\int \frac{\mathrm{d}y}{g(y)} = \int \mathrm{d}x$ leads to an implicit solution for $y$.

# Separable

Last week

A direct first-order ODE for $y(x)$ can be written in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x).$$

An autonomous first-order ODE for $y(x)$ can be written in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = g(y).$$

Today

A **separable first-order ODE** for $y(x)$ can be written in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x) \cdot g(y).$$

# Separable

An autonomous first-order ODE for $y(x)$ can be written in the form

$$\frac{dy}{dx} = g(y).$$

A **separable first-order ODE** for $y(x)$ can be written in the form

$$\frac{dy}{dx} = f(x) \cdot g(y).$$

Question: Is *every* autonomous ODE also separable?

# Solving a separable ODE

Example: $y' = \sin(x)\, y^2$

# Solving a separable IVP

Example: $y' = 10xy^{2/3}, y(2) = 8$

# Slope fields

Finding explicit solutions for other ODEs is often difficult (sometimes even impossible), but there is a way we can start to analyze solutions to *any* first-order ODE.

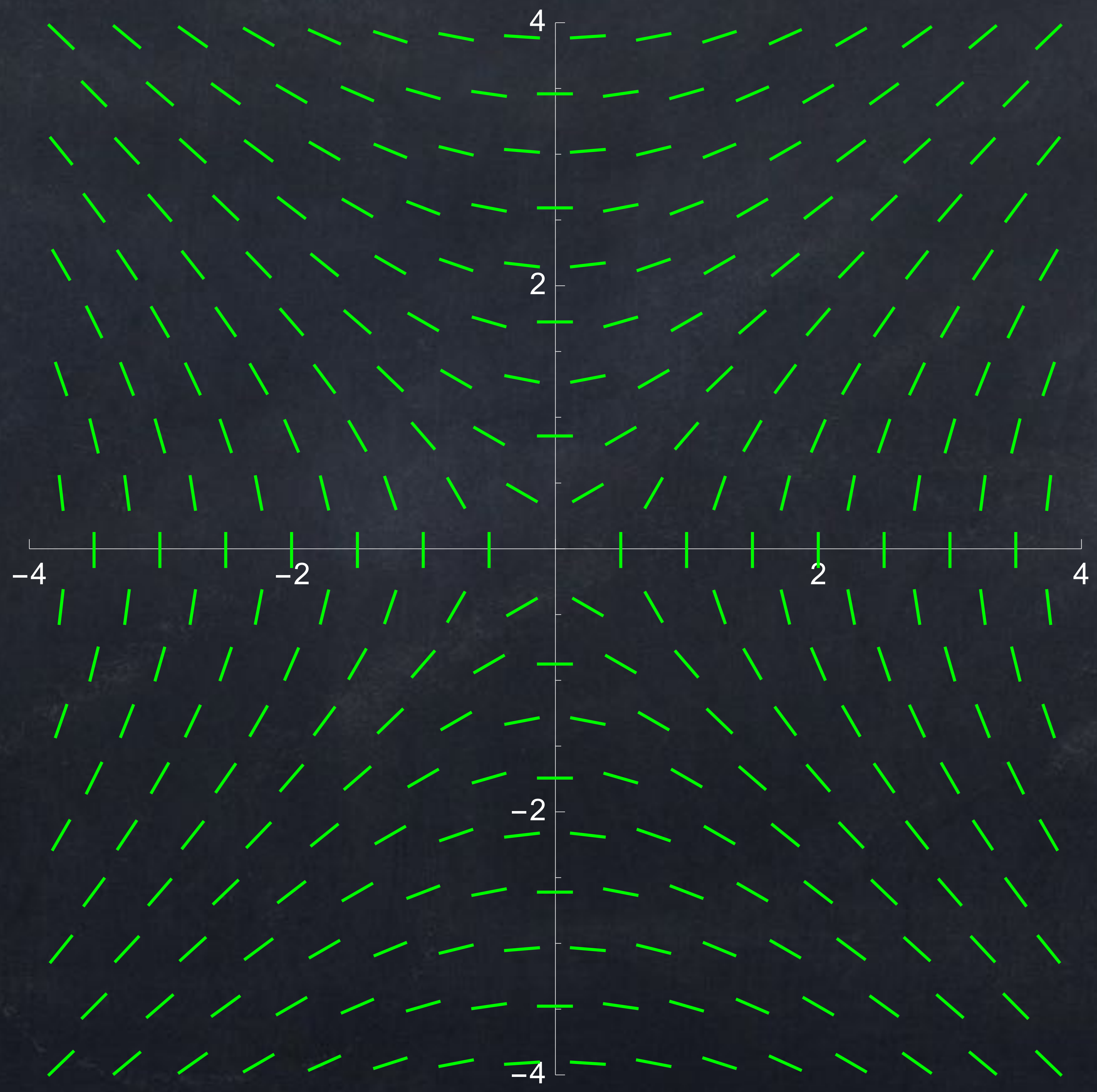
A **slope field** (also called a **direction field**) for a first-order ODE is a picture that shows the slope of the function at many different points.

It's easiest to understand using an example. Let's look at

$$y'(x) = \frac{x}{y}.$$

# Slope fields

The slope field for $y'(x) = \dfrac{x}{y}$ has a small line of slope $\dfrac{x}{y}$ at many points $(x, y)$.



Draw a slope field for $\dfrac{\mathrm{d}y}{\mathrm{d}x} = y$.

What “trivial solutions” (constant functions) does $\frac{\mathrm{d}y}{\mathrm{d}x} = 2 + y - y^2$ have?

Solve the ODE $x\,y' = \ln(x)\,y^3$.

# Types of ODEs

There are many names for categories or patterns of ODEs.

- Direct: $y' = g(x)$
- Autonomous: $y' = h(y)$
- Separable: $y' = g(x) \cdot h(y)$
- First-order linear: $y' = P(x)\,y + Q(x)$ NEW!

Remember that we can use other letters. $\frac{\mathrm{d}x}{\mathrm{d}t} = f(t)x + g(t)$ is also first-order linear.

Before we move on to “linear” ODEs, there is one specific example of a separable (actually, autonomous) equation that is worth looking at.